

Specification of Source §2 Stepper—2022 edition

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Reduction

The *reducer* $\Rightarrow$ is a partial function from programs/statements/expressions to programs/statements/expressions (with slight abuse of notation via overloading) and $\Rightarrow^*$ is its reflexive transitive closure. A *reduction* is a sequence of programs $p_1 \Rightarrow \dots \Rightarrow p_n$, where p_n is not reducible, i.e. there is no program q such that $p_n \Rightarrow q$. Here, the program p_n is called the *result of reducing* p_1 .

If p_n , the result of reducing a program, is of the form v ;, we call v the result of the program evaluation. If p_n is the empty sequence of statements, we declare the result of the program evaluation to be the value undefined. Reduction can get "stuck": the result can be a program that has neither of these two forms, which corresponds to a runtime error.

A *value* is a primitive number expression, primitive boolean expression, a primitive string expression, a function definition expression or a function declaration expression.

The *substitution* function $p[n := v]$ on programs/statements/expressions replaces every free occurrence of the name n in statement p by value v . Care must be taken to introduce and preserve co-references in this process; substitution can introduce cyclic references in the result of the substitution. For example, n may occur free in v , in which case every occurrence of n in p will be replaced by v such that n in v refers cyclically to the node at which the replacement happens.

Programs

Program-intro: In a sequence of statements, we can always reduce the first one that isn't a value statement.

$$\frac{\text{statement} \Rightarrow \text{statement}'}{[value;] \text{statement} \dots \Rightarrow [value;] \text{statement}' \dots}$$

Program-reduce: When the first two statements in a program are value statements, the first one can be removed.

$$\frac{v1, v2 \text{ are values}}{v1; v2; \text{statement} \dots \Rightarrow v2; \text{statement} \dots}$$

Eliminate-function-declaration: Function declarations as the first statement optionally after one value statement are substituted in the remaining statements.

$$\frac{f = \mathbf{function} \text{ name } (\text{parameters}) \text{ block}}{[value;] f \text{ statement} \dots \Rightarrow [value;] \text{statement} \dots [name := f]}$$

Eliminate-constant-declaration: Constant declarations as the first statement optionally after one value statement are substituted in the remaining statements.

$$\frac{c = \mathbf{const} \text{ name } = v}{[value;] c \text{ statement} \dots \Rightarrow [value;] \text{statement} \dots [name := v]}$$

Statements: Expression statements

Expression-statement-reduce: An expression statement is reducible if its expression is reducible.

$$\frac{e \Rightarrow e'}{e; \Rightarrow e';}$$

Statements: Constant declarations

Evaluate-constant-declaration: The right-hand expressions in constant declarations are evaluated.

$$\frac{\text{expression} \Rightarrow \text{expression}'}{\text{const name} = \text{expression} \Rightarrow \text{const name} = \text{expression}'}$$

Statements: Conditionals

Conditional-statement-predicate: A conditional statement is reducible if its predicate is reducible.

$$\frac{e \Rightarrow e'}{\text{if } (e) \{ \dots \} \text{ else } \{ \dots \} \Rightarrow \text{if } (e') \{ \dots \} \text{ else } \{ \dots \}}$$

SICP-JS Exercise 4.8 specifies that a conditional statement where the taken branch is non-value-producing is value-producing, with its value being **undefined**.

Conditional-statement-consequent: Immediately within a program or block statement, a conditional statement whose predicate is true reduces to the consequent block.

$$\text{if } (\text{true}) \{ \text{program}_1 \} \text{ else } \{ \text{program}_2 \} \Rightarrow \{ \text{undefined}; \text{program}_1 \}$$

Conditional-statement-alternative: Immediately within a program or block statement, a conditional statement whose predicate is false reduces to the alternative block.

$$\text{if } (\text{false}) \{ \text{program}_1 \} \text{ else } \{ \text{program}_2 \} \Rightarrow \{ \text{undefined}; \text{program}_2 \}$$

We can remove the **undefined**; if the conditional statement is in a block expression without affecting the return value of the block expression. This is performed to reduce the surprise factor.

Conditional-statement-blockexpr-consequent: Immediately within a block expression, a conditional statement whose predicate is true reduces to the consequent block.

$$\{ [value;] \text{if } (\text{true}) \{ \text{program}_1 \} \text{ else } \{ \text{program}_2 \} \} \Rightarrow \{ [value;] \{ \text{program}_1 \} \}$$

Conditional-statement-blockexpr-alternative: Immediately within a block expression, a conditional statement whose predicate is false reduces to the alternative block.

$$\{ [value;] \text{if } (\text{false}) \{ \text{program}_1 \} \text{ else } \{ \text{program}_2 \} \} \Rightarrow \{ [value;] \{ \text{program}_2 \} \}$$

Statements: Blocks

Block-statement-intro: A block statement is reducible if its body is reducible.

$$\frac{\text{program} \Rightarrow \text{program}'}{\{ \text{program} \} \Rightarrow \{ \text{program}' \}}$$

Block-statement-single-reduce: A block statement consisting of a single value statement reduces to that value statement.

$$\frac{}{\{value;\} \Rightarrow value;}$$

Block-statement-empty-reduce: An empty block statement (or a non-value-producing one, which will reduce to the empty block statement) is non-value-producing, i.e. reduces to ε .

$$\frac{}{\{\} \Rightarrow \varepsilon}$$

Expressions: Blocks

Block-expression-intro: A block expression is reducible if its body is reducible.

$$\frac{program \Rightarrow program'}{\{ program \} \Rightarrow \{ program' \}}$$

Block-expression-single-reduce: A block expression consisting of a single value statement reduces to **undefined**.

$$\frac{}{\{value;\} \Rightarrow \text{undefined}}$$

Block-expression-empty-reduce: An empty block expression reduces to **undefined**.

$$\frac{}{\{\} \Rightarrow \text{undefined}}$$

Block-expression-return-reduce-1: In a block expression starting with a value statement and then a return statement, the value statement can be removed.

$$\frac{}{\{value;\ \text{return}\ return-expression;\ statement...\} \Rightarrow \{\text{return}\ return-expression;\ statement...\}}$$

Block-expression-return-reduce-2: A block expression starting with a return statement reduces to the expression of the return statement.

$$\frac{}{\{\text{return}\ return-expression;\ statement...\} \Rightarrow return-expression}$$

Block expressions are currently used only as expanded forms of functions.

Expressions: Binary operators

Left-binary-reduce: An expression with binary operator can be reduced if its left sub-expression can be reduced.

$$\frac{e_1 \Rightarrow e'_1}{e_1\ binary-operator\ e_2 \Rightarrow e'_1\ binary-operator\ e_2}$$

And-shortcut-false: An expression with binary operator **&&** whose left sub-expression is **false** can be reduced to **false**.

$$\frac{}{\text{false} \ \&\&\ e \Rightarrow \text{false}}$$

And-shortcut-true: An expression with binary operator **&&** whose left sub-expression is **true** can be reduced to the right sub-expression.

$$\frac{}{\text{true} \ \&\&\ e \Rightarrow e}$$

Or-shortcut-true: An expression with binary operator `||` whose left sub-expression is **true** can be reduced to **true**.

$$\frac{}{\mathbf{true} \ || \ e \Rightarrow \mathbf{true}}$$

Or-shortcut-false: An expression with binary operator `||` whose left sub-expression is **false** can be reduced to the right sub-expression.

$$\frac{}{\mathbf{false} \ || \ e \Rightarrow e}$$

Right-binary-reduce: An expression with binary operator can be reduced if its left sub-expression is a value and its right sub-expression can be reduced.

$$\frac{e_2 \Rightarrow e'_2, \text{ and } \textit{binary-operator} \text{ is not } \&\& \text{ or } ||}{v \ \textit{binary-operator} \ e_2 \Rightarrow v \ \textit{binary-operator} \ e'_2}$$

Prim-binary-reduce: An expression with binary operator can be reduced if its left and right sub-expressions are values and the corresponding function is defined for those values.

$$\frac{v \text{ is result of } v_1 \ \textit{binary-operator} \ v_2}{v_1 \ \textit{binary-operator} \ v_2 \Rightarrow v}$$

Expressions: Unary operators

Unary-reduce: An expression with unary operator can be reduced if its sub-expression can be reduced.

$$\frac{e \Rightarrow e'}{\textit{unary-operator} \ e \Rightarrow \textit{unary-operator} \ e'}$$

Prim-unary-reduce: An expression with unary operator can be reduced if its sub-expression is a value and the corresponding function is defined for that value.

$$\frac{v' \text{ is result of } \textit{unary-operator} \ v}{\textit{unary-operator} \ v \Rightarrow v'}$$

Expressions: conditionals

Conditional-predicate-reduce: A conditional expression can be reduced, if its predicate can be reduced.

$$\frac{e_1 \Rightarrow e'_1}{e_1 \ ? \ e_2 : e_3 \Rightarrow e'_1 \ ? \ e_2 : e_3}$$

Conditional-true-reduce: A conditional expression whose predicate is the value **true** can be reduced to its consequent expression.

$$\frac{}{\mathbf{true} \ ? \ e_1 : e_2 \Rightarrow e_1}$$

Conditional-false-reduce: A conditional expression whose predicate is the value **false** can be reduced to its alternative expression.

$$\frac{}{\mathbf{false} \ ? \ e_1 : e_2 \Rightarrow e_2}$$

Expressions: function application

Application-functor-reduce: A function application can be reduced if its functor expression can be reduced.

$$\frac{e \Rightarrow e'}{e (\text{expressions}) \Rightarrow e' (\text{expressions})}$$

Application-argument-reduce: A function application can be reduced if one of its argument expressions can be reduced and all preceding arguments are values.

$$\frac{e \Rightarrow e'}{v (v_1 \dots v_i e \dots) \Rightarrow v (v_1 \dots v_i e' \dots)}$$

Function-declaration-application-reduce: The application of a function declaration can be reduced, if all arguments are values.

$$\frac{f = \mathbf{function} \ n (x_1 \dots x_n) \ block}{f (v_1 \dots v_n) \Rightarrow block[x_1 := v_1] \dots [x_n := v_n][n := f]}$$

Function-definition-application-reduce: The application of a function definition can be reduced, if all arguments are values.

$$\frac{f = (x_1 \dots x_n) \Rightarrow b, \text{ where } b \text{ is an expression or block}}{f (v_1 \dots v_n) \Rightarrow b[x_1 := v_1] \dots [x_n := v_n]}$$

Substitution

Identifier: An identifier with the same name as x is substituted with e_x .

$$\frac{}{x[x := e_x] = e_x}$$

$$\frac{name \neq x}{name[x := e_x] = name}$$

Expression statement: All occurrences of x in e are substituted with e_x .

$$\frac{}{e; [x := e_x] = e[x := e_x];}$$

Binary expression: All occurrences of x in the operands are substituted with e_x .

$$\frac{}{(e_1 \text{ binary-operator } e_2)[x := e_x] = e_1[x := e_x] \text{ binary-operator } e_2[x := e_x]}$$

Unary expression: All occurrences of x in the operand are substituted with e_x .

$$\frac{}{(unary-operator \ e)[x := e_x] = unary-operator \ e[x := e_x]}$$

Conditional expression: All occurrences of x in the operands are substituted with e_x .

$$\frac{}{(e_1 \ ? \ e_2 : e_3)[x := e_x] = e_1[x := e_x] \ ? \ e_2[x := e_x] : e_3[x := e_x]}$$

Logical expression: All occurrences of x in the operands are substituted with e_x .

$$\frac{}{(e_1 \ || \ e_2)[x := e_x] = e_1[x := e_x] \ || \ e_2[x := e_x]}$$

$$\frac{}{(e_1 \ \&\& \ e_2)[x := e_x] = e_1[x := e_x] \ \&\& \ e_2[x := e_x]}$$

Call expression: All occurrences of x in the arguments and the function expression of the application e are substituted with e_x .

$$\frac{}{(e \ (\ x_1 \dots x_n \))[x := e_x] = e[x := e_x] \ (\ x_1[x := e_x] \dots x_n[x := e_x] \)}$$

Function declaration: All occurrences of x in the body of a function are substituted with e_x under given circumstances.

1. Function declaration where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \dots, n\} \text{ s.t. } x = x_i}{(\text{function name } (\ x_1 \dots x_n \) \ \text{block})[x := e_x] = \text{function name } (\ x_1 \dots x_n \) \ \text{block}}$$

2. Function declaration where x does not have the same name as a parameter.

(a) No parameter of the function occurs free in e_x .

$$\frac{\forall i \in \{1, \dots, n\} \text{ s.t. } x \neq x_i, \forall j \in \{1, \dots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{(\text{function name } (x_1 \dots x_n) \text{ block})[x := e_x]} = \text{function name } (x_1 \dots x_n) \text{ block}[x := e_x]$$

(b) A parameter of the function occurs free in e_x .

$$\frac{\forall i \in \{1, \dots, n\} \text{ s.t. } x \neq x_i, \exists j \in \{1, \dots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{(\text{function name } (x_1 \dots x_j \dots x_n) \text{ block})[x := e_x]} = (\text{function name } (x_1 \dots y \dots x_n) \text{ block}[x_j := y])[x := e_x]$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the function declaration that clash with variables that occur free in e_x , at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x .

Lambda expression: All occurrences of x in the body of a lambda expression are substituted with e_x under given circumstances.

1. Lambda expression where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \dots, n\} \text{ s.t. } x = x_i}{((x_1 \dots x_n) \Rightarrow \text{block})[x := e_x]} = ((x_1 \dots x_n) \Rightarrow \text{block})$$

2. Lambda expression where x does not have the same name as a parameter.

(a) No parameter of the lambda expression occurs free in e_x .

$$\frac{\forall i \in \{1, \dots, n\} \text{ s.t. } x \neq x_i, \forall j \in \{1, \dots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{((x_1 \dots x_n) \Rightarrow \text{block})[x := e_x]} = ((x_1 \dots x_n) \Rightarrow \text{block}[x := e_x])$$

(b) A parameter of the lambda expression occurs free in e_x .

$$\frac{\forall i \in \{1, \dots, n\} \text{ s.t. } x \neq x_i, \exists j \in \{1, \dots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{((x_1 \dots x_j \dots x_n) \Rightarrow \text{block})[x := e_x]} = ((x_1 \dots y \dots x_n) \Rightarrow \text{block}[x_j := y])[x := e_x]$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the lambda expression that clash with variables that occur free in e_x , at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x .

Block expression: All occurrences of x in the statements of a block expression are substituted with e_x under given circumstances.

1. Block expression in which x is declared.

$$\frac{x \text{ is declared in } block}{block[x := e_x] = block}$$

2. Block expression in which x is not declared.

- (a) No names declared in the block occurs free in e_x .

$$\frac{x \text{ is not declared in } block, \text{ name declared in } block \text{ does not occur free in } e_x}{block[x := e_x] = [block[0][x := e_x], \dots, block[n][x := e_x]]}$$

- (b) A name declared in the block occurs free in e_x .

$$\frac{x \text{ is not declared in } block, \text{ name declared in } block \text{ occurs free in } e_x}{block[x := e_x] = [block[0][name := y], \dots, block[n][name := y]][x := e_x]}$$

Substitution is applied to the whole expression again as to recursively detect and re-name all declared names of the block expression that clash with variables that occur free in e_x , at which point (i) takes place. Note that the name y is not declared in, nor occurs in $block$ and e_x .

Variable declaration: All occurrences of x in the declarators of a variable declaration are substituted with e_x .

$$\overline{declarations[x := e_x] = [declarations[0][x := e_x] \dots declarations[n][x := e_x]]}$$

Return statement: All occurrences of x in the expression that is to be returned are substituted with e_x .

$$\overline{(\mathbf{return} \ e;)[x := e_x] = \mathbf{return} \ e[x := e_x];}$$

Conditional statement: All occurrences of x in the condition, consequent, and alternative expressions of a conditional statement are substituted with e_x .

$$\overline{(\mathbf{if} \ (e) \ block \ \mathbf{else} \ block)[x := e_x] = \mathbf{if} \ (e[x := e_x]) \ block[x := e_x] \ \mathbf{else} \ block[x := e_x]}$$

Array expression: All occurrences of x in the elements of an array are substituted with e_x .

$$\overline{[x_1, \dots, x_n][x := e_x] = [x_1[x := e_x], \dots, x_n[x := e_x]]}$$

Free names

Let $\triangleright$ be the relation that defines the set of free names of a given Source expression; the symbols p_1 and p_2 shall henceforth refer to unary and binary operations, respectively. That is, p_1 ranges over $\{!\}$ and p_2 ranges over $\{||, \&\&, +, -, *, /, ==, >, <\}$.

Identifier:

$$\frac{}{x \triangleright \{x\}}$$

$$\frac{}{name \triangleright \emptyset}$$

Boolean:

$$\frac{}{\mathbf{true} \triangleright \emptyset}$$

$$\frac{}{\mathbf{false} \triangleright \emptyset}$$

Expression statement:

$$\frac{e \triangleright S}{e; \triangleright S}$$

Unary expression:

$$\frac{e \triangleright S}{p_1(e) \triangleright S}$$

Binary expression:

$$\frac{e_1 \triangleright S_1, e_2 \triangleright S_2}{p_2(e_1, e_2) \triangleright S_1 \cup S_2}$$

Conditional expression:

$$\frac{e_1 \triangleright S_1, e_2 \triangleright S_2, e_3 \triangleright S_3}{e_1 ? e_2 : e_3 \triangleright S_1 \cup S_2 \cup S_3}$$

Call expression:

$$\frac{e \triangleright S, e_k \triangleright T_k}{e(e_1, \dots, e_n) \triangleright S \cup T_1 \cup \dots \cup T_n}$$

Function declaration:

$$\frac{block \triangleright S}{\mathbf{function\ name\ } (x_1 \dots x_n) \ block \triangleright S - \{x_1, \dots, x_n\}}$$

Lambda expression:

$$\frac{block \triangleright S}{(x_1 \dots x_n) \Rightarrow block \triangleright S - \{x_1, \dots, x_n\}}$$

Block expression:

$$\frac{block[k] \triangleright S_k, \text{ } T \text{ contains all names declared in } block}{block \triangleright (S_1 \cup \dots \cup S_n) - T}$$

Constant declaration:

$$\frac{e \triangleright S}{\mathbf{const\ name = e; } \triangleright S}$$

Return statement:

$$\frac{e \triangleright S}{\mathbf{return\ e; } \triangleright S}$$

Conditional statement:

$$\frac{e \triangleright S, \text{ } block_1 \triangleright T_1, \text{ } block_2 \triangleright T_2}{\mathbf{if\ (e)\ block_1\ else\ block_2 } \triangleright S \cup T_1 \cup T_2}$$