Specification of Source §2 Stepper—2021 edition

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October 14, 2021

Reduction

The *reducer* $\Rightarrow$ is a partial function from programs/statements/expressions to programs/statements/expressions (with slight abuse of notation via overloading) and $\Rightarrow^*$ is its reflexive transitive closure. A *reduction* is a sequence of programs $p_1 \Rightarrow \cdots \Rightarrow p_n$, where p_n is not reducible, i.e. there is no program q such that $p_n \Rightarrow q$. Here, the program p_n is called the *result of reducing* p_1.
A *value* is a primitive number expression, primitive boolean expression, a primitive string expression, a function definition expression or a function declaration statement.
The *substitution* function $p[n := v]$ on programs/statements/expressions replaces every free occurrence of the name n in statement p by value v. Care must be taken to introduce and preserve co-references in this process; substitution can introduce cyclic references in the result of the substitution. For example, n may occur free in v, in which case every occurrence of n in p will be replaced by v such that n in v refers cyclically to the node at which the replacement happens.

Programs

First-statement: In a sequence of statements, we can always reduce the first one.

$$\frac{\mathit{statement} \;\Rightarrow\; \mathit{statement}'}{\mathit{statement}\ldots \Rightarrow \mathit{statement}'\ldots}$$

Eliminate-function-declaration: Function declarations as first statements are substituted in the remaining statements.

$$\frac{f = \texttt{function}\ \mathit{name}\ \texttt{(}\ \mathit{parameters}\ \texttt{)}\ \mathit{block}}{f\ \mathit{statement}\ldots \;\Rightarrow\; \mathit{statement}\ldots[\mathit{name} := f]}$$

Eliminate-constant-declaration: Constant declarations as first statements are substituted in the remaining statements.

$$\frac{c = \texttt{const}\ \mathit{name}\ \texttt{=}\ v}{c\ \mathit{statement}\ldots \;\Rightarrow\; \mathit{statement}\ldots[\mathit{name} := v]}$$

Eliminate-Values: Values as first statemments are discarded, if they are preceding one or more statements in a statement sequence.

$$\frac{v \text{ is a value}}{v\texttt{;}\ \mathit{statement}+ \;\Rightarrow\; \mathit{statement}+}$$

Statements: Constant declarations

Evaluate-constant-declaration: The right-hand expressions in constant declarations are evaluated.

$$\frac{\textit{expression} \Rightarrow \textit{expression}'}{\texttt{const}\ \textit{name}\ \texttt{=}\ \textit{expression} \Rightarrow \texttt{const}\ \textit{name}\ \texttt{=}\ \textit{expression}'}$$

Statements: Conditionals

Conditional-statement-predicate: A conditional statement is reducible if its predicate is reducible.

$$\frac{e \Rightarrow e'}{\texttt{if (}\ e\ \texttt{) \{}\ \cdots\ \texttt{\} else \{}\ \cdots\ \texttt{\}} \Rightarrow \texttt{if (}\ e'\ \texttt{) \{}\ \cdots\ \texttt{\} else \{}\ \cdots\ \texttt{\}}}$$

Conditional-statement-consequent: A conditional statement whose predicate is true reduces to the consequent block.

$$\frac{}{\texttt{if (true) \{}\ \textit{statement}_1\ \texttt{\} else \{}\ \textit{statement}_2\ \texttt{\}} \Rightarrow \texttt{\{}\ \textit{statement}_1\ \texttt{\}}}$$

Conditional-statement-alternative: A conditional statement whose predicate is false reduces to the alternative block.

$$\frac{}{\texttt{if (false) \{}\ \textit{statement}_1\ \texttt{\} else \{}\ \textit{statement}_2\ \texttt{\}} \Rightarrow \texttt{\{}\ \textit{statement}_2\ \texttt{\}}}$$

Statements: Blocks

Block-statement-reduce: A block statement is reducible if its program is reducible.

$$\frac{\textit{program} \Rightarrow \textit{program}'}{\texttt{\{}\ \textit{program}\ \texttt{\}} \Rightarrow \texttt{\{}\ \textit{program}'\ \texttt{\}}}$$

Block-statement-undefined: A block statement whose body only contains a single value statement reduces to the value `undefined`.

$$\frac{}{\texttt{\{}\ v\ \texttt{;\ \}} \Rightarrow \texttt{undefined}}$$

Block-statement-reduce-return: A block statement whose body only contains a single return statement can be reduced by reducing the return expression.

$$\frac{e \Rightarrow e'}{\texttt{\{ return}\ e\ \texttt{;\ \}} \Rightarrow \texttt{\{ return}\ e'\ \texttt{;\ \}}}$$

Block-statement-eliminate-return: A block statement whose body only contains a single return value reduces to that value.

$$\frac{}{\texttt{\{ return}\ v\ \texttt{;\ \}} \Rightarrow v}$$

Statements: Expression statements

Expression-statement-reduce: An expression statement is reducible if its expression is reducible.

$$\frac{e \Rightarrow e'}{e\texttt{;} \Rightarrow e'\texttt{;}}$$

Expressions: Binary operators

Left-binary-reduce: An expression with binary operator can be reduced if its left sub-expression can be reduced.

$$\frac{e_1 \;\Rightarrow\; e_1'}{e_1 \textit{ binary-operator } e_2 \;\Rightarrow\; e_1' \textit{ binary-operator } e_2}$$

And-shortcut-false: An expression with binary operator **`&&`** whose left sub-expression is **`false`** can be reduced to **`false`**.

$$\frac{}{\texttt{false \&\&}\ e \;\Rightarrow\; \texttt{false}}$$

And-shortcut-true: An expression with binary operator **`&&`** whose left sub-expression is **`true`** can be reduced to the right sub-expression.

$$\frac{}{\texttt{true \&\&}\ e \;\Rightarrow\; e}$$

Or-shortcut-true: An expression with binary operator **`||`** whose left sub-expression is **`true`** can be reduced to **`true`**.

$$\frac{}{\texttt{true ||}\ e \;\Rightarrow\; \texttt{true}}$$

Or-shortcut-false: An expression with binary operator **`||`** whose left sub-expression is **`false`** can be reduced to the right sub-expression.

$$\frac{}{\texttt{false ||}\ e \;\Rightarrow\; e}$$

Right-binary-reduce: An expression with binary operator can be reduced if its left sub-expression is a value and its right sub-expression can be reduced.

$$\frac{e_2 \;\Rightarrow\; e_2', \text{and } \textit{binary-operator} \text{is not } \texttt{\&\&} \text{ or } \texttt{||}}{v \textit{ binary-operator } e_2 \;\Rightarrow\; v \textit{ binary-operator } e_2'}$$

Prim-binary-reduce: An expression with binary operator can be reduced if its left and right sub-expressions are values and the corresponding function is defined for those values.

$$\frac{v \text{ is result of } v_1 \textit{ binary-operator } v_2}{v_1 \textit{ binary-operator } v_2 \;\Rightarrow\; v}$$

Expressions: Unary operators

Unary-reduce: An expression with unary operator can be reduced if its sub-expression can be reduced.

$$\frac{e \;\Rightarrow\; e'}{\textit{unary-operator } e \;\Rightarrow\; \textit{unary-operator } e'}$$

Prim-unary-reduce: An expression with unary operator can be reduced if its sub-expression is a value and the corresponding function is defined for that value.

$$\frac{v' \text{ is result of } \textit{unary-operator } v}{\textit{unary-operator } v \;\Rightarrow\; v'}$$

Expressions: conditionals

Conditional-predicate-reduce: A conditional expression can be reduced, if its predicate can be reduced.

$$\frac{e_1 \;\Rightarrow\; e_1'}{e_1 \texttt{ ? } e_2 \texttt{ : } e_3 \;\Rightarrow\; e_1' \texttt{ ? } e_2 \texttt{ : } e_3}$$

Conditional-true-reduce: A conditional expression whose predicate is the value **`true`** can be reduced to its consequent expression.

$$\frac{}{\texttt{true ? } e_1 \texttt{ : } e_2 \;\Rightarrow\; e_1}$$

Conditional-false-reduce: A conditional expression whose predicate is the value **`false`** can be reduced to its alternative expression.

$$\frac{}{\texttt{false ? } e_1 \texttt{ : } e_2 \;\Rightarrow\; e_2}$$

Expressions: function application

Application-functor-reduce: A function application can be reduced if its functor expression can be reduced.

$$\frac{e \;\Rightarrow\; e'}{e \texttt{ (} \textit{expressions} \texttt{)} \;\Rightarrow\; e' \texttt{ (} \textit{expressions} \texttt{)}}$$

Application-argument-reduce: A function application can be reduced if one of its argument expressions can be reduced and all preceding arguments are values.

$$\frac{e \;\Rightarrow\; e'}{v \texttt{ (} v_1 \ldots v_i \; e \ldots \texttt{)} \;\Rightarrow\; v \texttt{ (} v_1 \ldots v_i \; e' \ldots \texttt{)}}$$

Function-declaration-application-reduce: The application of a function declaration can be reduced, if all arguments are values.

$$\frac{f = \texttt{function } n \texttt{ (} x_1 \ldots x_n \texttt{)} \; \textit{block}}{f \texttt{ (} v_1 \ldots v_n \texttt{)} \;\Rightarrow\; \textit{block}[x_1 := v_1] \ldots [x_n := v_n][n := f]}$$

Function-definition-application-reduce: The application of a function definition can be reduced, if all arguments are values.

$$\frac{f = \texttt{ (} x_1 \ldots x_n \texttt{) => } b\text{, where } b \text{ is an expression or block}}{f \texttt{ (} v_1 \ldots v_n \texttt{)} \;\Rightarrow\; b[x_1 := v_1] \ldots [x_n := v_n]}$$

Substitution

Identifier: An identifier with the same name as x is substituted with e_x.

$$\overline{x[x := e_x] \;=\; e_x}$$

$$\frac{\mathit{name} \neq x}{\mathit{name}[x := e_x] \;=\; \mathit{name}}$$

Expression statement: All occurrences of x in e are substituted with e_x.

$$\overline{e\texttt{;}[x := e_x] \;=\; e\,[x := e_x]\texttt{;}}$$

Binary expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \;\mathit{binary\text{-}operator}\; e_2)[x := e_x] \;=\; e_1[x := e_x] \;\mathit{binary\text{-}operator}\; e_2[x := e_x]}$$

Unary expression: All occurrences of x in the operand are substituted with e_x.

$$\overline{(\mathit{unary\text{-}operator}\; e)[x := e_x] \;=\; \mathit{unary\text{-}operator}\; e[x := e_x]}$$

Conditional expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \;\texttt{?}\; e_2 \;\texttt{:}\; e_3)[x := e_x] \;=\; e_1[x := e_x] \;\texttt{?}\; e_2[x := e_x] \;\texttt{:}\; e_3[x := e_x]}$$

Logical expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \;\texttt{||}\; e_2)[x := e_x] \;=\; e_1[x := e_x] \;\texttt{||}\; e_2[x := e_x]}$$

$$\overline{(e_1 \;\texttt{\&\&}\; e_2)[x := e_x] \;=\; e_1[x := e_x] \;\texttt{\&\&}\; e_2[x := e_x]}$$

Call expression: All occurrences of x in the arguments and the function expression of the application e are substituted with e_x.

$$\overline{(e \;\texttt{(}\; x_1 \ldots x_n \;\texttt{)})[x := e_x] \;=\; e[x := e_x] \;\texttt{(}\; x_1[x := e_x] \ldots x_n[x := e_x] \;\texttt{)}}$$

Function declaration: All occurrences of x in the body of a function are substituted with e_x under given circumstances.

① Function declaration where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \cdots, n\} \text{ s.t. } x \ = \ x_i}{(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \ = \ \texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block}}$$

② Function declaration where x does not have the same name as a parameter.

(i) No parameter of the function occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \ \neq \ x_i, \ \forall j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{\begin{array}{c}(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \\ = \\ \texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block}[x := e_x]\end{array}}$$

(ii) A parameter of the function occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \ \neq \ x_i, \ \exists j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{\begin{array}{c}(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_j \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \\ = \\ (\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots y \ldots x_n\ \texttt{)}\ \textit{block}[x_j := y])[x := e_x]\end{array}}$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the function declaration that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Lambda expression: All occurrences of x in the body of a lambda expression are substituted with e_x under given circumstances.

① Lambda expression where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \cdots, n\} \text{ s.t. } x \ = \ x_i}{(\texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \ = \ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}}$$

② Lambda expression where x does not have the same name as a parameter.

(i) No parameter of the lambda expression occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \ \neq \ x_i, \ \forall j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{(\texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \ = \ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}[x := e_x]}$$

(ii) A parameter of the lambda expression occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \ \neq \ x_i, \ \exists j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{(\texttt{(}\ x_1 \ldots x_j \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \ = \ (\texttt{(}\ x_1 \ldots y \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}[x_j := y])[x := e_x]}$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the lambda expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Block expression: All occurrences of x in the statements of a block expression are substituted with e_x under given circumstances.

① Block expression in which x is declared.

$$\frac{x \text{ is declared in } \textit{block}}{\textit{block}[x := e_x] \;=\; \textit{block}}$$

② Block expression in which x is not declared.

(i) No names declared in the block occurs free in e_x.

$$\frac{x \text{ is not declared in } \textit{block},\;\; \textit{name} \text{ declared in } \textit{block} \text{ does not occur free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][x := e_x],\; \ldots,\; \textit{block}[n][x := e_x]]}$$

(ii) A name declared in the block occurs free in e_x.

$$\frac{x \text{ is not declared in } \textit{block},\;\; \textit{name} \text{ declared in } \textit{block} \text{ occurs free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][\textit{name} := y],\; \ldots,\; \textit{block}[n][\textit{name} := y]][x := e_x]}$$

Substitution is applied to the whole expression again as to recursively detect and rename all declared names of the block expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Variable declaration: All occurrences of x in the declarators of a variable declaration are substituted with e_x.

$$\overline{\textit{declarations}[x := e_x] \;=\; [\textit{declarations}[0][x := e_x] \ldots \textit{declarations}[n][x := e_x]]}$$

Return statement: All occurrences of x in the expression that is to be returned are substituted with e_x.

$$\overline{(\texttt{return}\ e\texttt{;})[x := e_x] \;=\; \texttt{return}\ e[x := e_x]\texttt{;}}$$

Conditional statement: All occurrences of x in the condition, consequent, and alternative expressions of a conditional statement are substituted with e_x.

$$\overline{(\texttt{if (}\ e\ \texttt{)}\ \textit{block}\ \texttt{else}\ \textit{block})[x := e_x] \;=\; \texttt{if (}\ e[x := e_x]\ \texttt{)}\ \textit{block}[x := e_x]\ \texttt{else}\ \textit{block}[x := e_x]}$$

Array expression: All occurrences of x in the elements of an array are substituted with e_x.

$$\overline{[x_1,\; \ldots,\; x_n][x := e_x] \;=\; [x_1[x := e_x],\; \ldots,\; x_n[x := e_x]]}$$

Free names

Let $\triangleright$ be the relation that defines the set of free names of a given Source expression; the symbols p_1 and p_2 shall henceforth refer to unary and binary operations, respectively. That is, p_1 ranges over {`!`} and p_2 ranges over {`||`, `&&`, `+`, `-`, `*`, `/`, `===`, `>`, `<`}.

Identifier:

$$\frac{}{x \ \triangleright \ \{x\}}$$

$$\frac{}{\textit{name} \ \triangleright \ \varnothing}$$

Boolean:

$$\frac{}{\texttt{true} \ \triangleright \ \varnothing}$$

$$\frac{}{\texttt{false} \ \triangleright \ \varnothing}$$

Expression statement:

$$\frac{e \ \triangleright \ S}{e\texttt{;} \ \triangleright \ S}$$

Unary expression:

$$\frac{e \ \triangleright \ S}{p_1(e) \ \triangleright \ S}$$

Binary expression:

$$\frac{e_1 \ \triangleright \ S_1, \ e_2 \ \triangleright \ S_2}{p_1(e_1, e_2) \ \triangleright \ S_1 \cup S_2}$$

Conditional expression:

$$\frac{e_1 \ \triangleright \ S_1, \ e_2 \ \triangleright \ S_2, \ e_3 \ \triangleright \ S_3}{e_1 \ \texttt{?} \ e_2 \ \texttt{:} \ e_3 \ \triangleright \ S_1 \cup S_2 \cup S_3}$$

Call expression:

$$\frac{e \ \triangleright \ S, \ e_k \ \triangleright \ T_k}{e(e_1, \ldots, e_n) \ \triangleright \ S \cup T_1 \cup \ldots \cup T_n}$$

Function declaration:

$$\frac{\textit{block} \ \triangleright \ S}{\texttt{function} \ \textit{name} \ \texttt{(} \ x_1 \ldots x_n \ \texttt{)} \ \textit{block} \ \triangleright \ S - \{x_1, \ldots, x_n\}}$$

Lambda expression:

$$\frac{\textit{block} \ \triangleright \ S}{\texttt{(} \ x_1 \ldots x_n \ \texttt{)} \ \texttt{=>} \ \textit{block} \ \triangleright \ S - \{x_1, \ldots, x_n\}}$$

Block expression:

$$\frac{\textit{block}[k] \ \triangleright \ S_k, \ T \text{ contains all names declared in } \textit{block}}{\textit{block} \ \triangleright \ (S_1 \cup \ldots \cup S_n) - T}$$

Constant declaration:

$$\frac{e \ \triangleright \ S}{\texttt{const } \textit{name} \texttt{ = } e\texttt{; } \ \triangleright \ S}$$

Return statement:

$$\frac{e \ \triangleright \ S}{\texttt{return } e\texttt{; } \ \triangleright \ S}$$

Conditional statement:

$$\frac{e \ \triangleright \ S, \ \textit{block}_1 \ \triangleright \ T_1, \ \textit{block}_2 \ \triangleright \ T_2}{\texttt{if (} e \texttt{) } \textit{block}_1 \texttt{ else } \textit{block}_2 \ \triangleright \ S \cup T_1 \cup T_2}$$